SOLVING THE MINMAX PRODUCT RATE VARIATION PROBLEM (PRVP) AS A BOTTLENECK ASSIGNMENT PROBLEM

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ABSTRACT

The minmax PRVP consists in sequencing different type units minimizing the maximum value of a discrepancy function between ideal and actual production rates. One means of solution lies in its reduction to a Bottleneck Assignment Problem (BAP) with a matrix of a special structure. Three different approaches which use some specific BAP algorithms and other that take into account the PRVP matrix properties have been adapted to solve it. The work presents a computational experience with symmetric and asymmetric objective functions and recommendations about the most efficient way to find optimal solutions.

Keywords: Product Rate Variation Problem, Bottleneck Assignment Problem, Assignment Problem, Matching.

AMS Classification: 90B35, 90C47, 90C08.

1. Introduction

The PRVP is an important production problem that arises on mixed-model assembly lines. Consider that there are V variants or types to be produced on an assembly line that needs negligible time to change from one variant to another. Assume that each unit, regardless of its corresponding variant, requires the same production time (the cycle time of the line, which can be adopted as the unit of time without loss of generality).
There are $U$ units to be produced, of which $u_i$ are of type $i$ $(i=1,...,V)$ with $\sum_{i=1}^{V} u_i = U$, so that production rates of different product types will be kept as constant as possible. The time horizon is of $U$ time units, where one copy of product $i$, $i=1,...,V$ will be produced in each time period. The ideal production rate $r_i$ of each variant can be calculated as:

$$r_i = \frac{u_i}{U}, \quad i = 1,...,V$$

On other hand the problem objective can be considered as the determination of regular sequences in order to minimize the total variation in the production rates over time for different variants involved.

The sequence can be described by means of $x_{ih}$ values (the total production of product $i$ in time periods from $1$ to $h$ inclusive, $h=1,...,U$) so, in order to evaluate the deviation between actual $x_{ih}$ and ideal productions, discrepancy functions $f_i(x_{ih},h)$ can be introduced. The total variation in product rates can be quantified minimizing the maximum value of discrepancy function or functions. We will call such a problem the minmax PRVP. The problem can be formalized as:

$$\min z_m = \max_{1 \leq i \leq V} \max_{1 \leq h \leq U} f_i(x_{ih}, h)$$

The main objective to be achieved, solving the minmax PRVP, is obtaining a schedule that enables us to attenuate undesirable, too large deviations at every time period of a total production time horizon. One means of solution lies in its reduction to a Bottleneck Assignment Problem (BAP) and the application of specific algorithms to it. Such a reduction allows us to use symmetric and asymmetric objective functions.

In industrial practice it is very usual to determine schedules that could correspond to thousands of units of different types to be sequenced. Therefore the problem with a large number of total product units to be produced and a large number of product types (large dimension problem) is of special interest.

This work presents, for the first time, the exploration and the development of specific algorithms taking into account special properties of the minmax PRVP considered as a BAP, which allows us to obtain more efficiency of algorithms, and a computational experience of solving the aforementioned problem for instances of large dimensions with symmetric and asymmetric objective functions.

In Section 2 the current state of the problem is exposed; in Section 3 the minmax PRVP as a Bottleneck Assignment Problem (BAP) with specific properties is solved and computational results are reported; in Section 4 some conclusions are drawn.

## 2. The state of the art of the problem

Different authors have dealt with the minmax PRVP. Steiner and Yeomans (1993) introduce a new non-convex objective function which is the maximal deviation between the actual production and the ideal production.
They show that a schedule always exists such that the deviation of actual production from the ideal for all product types is never greater than one unit. The authors develop an optimization procedure for the minmax PRVP considering it as a matching problem in a bipartite graph and demonstrate that target value $T$ belongs to the interval $[1-r_{max},1]$. The conjecture of reducing the minmax PRVP to a BAP was first mentioned in Kubiak (1993). Bautista et al. (1997) demonstrate such a reduction and devise a matrix computing method, used in this research. Brauner and Crama (2001) establish some new bounds on the optimal target value. One of them is the upper bound $T_{opt} \leq 1 - \frac{1}{U}$.

Kovalyov et al. (2001) present computational results of the minmax PRVP solution by means of solving the sequence of matching problems. They have restricted their computational experience to dimension of 200.

This work deals with the minmax PRVP regarded as a Bottleneck Assignment Problem (BAP). To save memory and computational time in our experiments we have used the band which contains only positions in which the following is fulfilled: $|x_{ih} - r_i \cdot h| < 1$ (it is called as a “quota” band) and two aforementioned bounds on the target value. In the case of asymmetric discrepancy functions we have introduced an extension of the band that does not have a symmetric property as the “quota” band.

## 3. The minmax PRVP as a BAP

### 3.1 Algorithms for the BAP

In this research three main approaches have been adapted for the minmax PRVP solution as a BAP. They are:

1. Solving the problem by means of specific BAP algorithms.

2. Solving the problem as a sequence of assignment problems (AP).

3. Solving the problem as a sequence of matching problems.

The first method requires the use of specific algorithms to solve the BAP. Two of the available specific codes for solving a BAP have been involved in our study. They are BASS (Carpaneto and Toth (1981)) and BOTJV (Jonker and Volgenant (1987)).

The second is based on the one presented in Woolsey and Swanson (1975) and consists in the solution of a sequence of AP using available specific algorithms for this (such as NAUC, APC, LAPJV and LAPJVsp described, for example, in the study on the AP by Dell’Amico and Toth (1998)). Two specific modifications to the approach have been introduced. There are: an introduction of a binary matrix instead of the original, Bottleneck assignment matrix, and an application of bisection search technique to the feasible interval for a target value in order to obtain an optimal one.
Another way of looking at the assignment problem (AP) (third approach) is its consideration on a bipartite graph (Steiner and Yeomans (1993)). This approach involves an application of a Bipartite Maximum Cardinality Matching algorithm (BMCM). A new element we have introduced with attempt to improve computational results consists in starting with a heuristic PRVP solution of a good quality given by one of the well approximating heuristic algorithms (for example, Webster heuristic). So to solve the problem the BMCM is applied to a bipartite graph considered for the set of edges entering into “quota” band, which can start with complete initial matching or almost complete. We have implemented this approach with bisection search (proposed by Steiner and Yeomans (1993)) and without its application. We have named all approaches and their modifications for the ease of dealing with them. Figure 1 presents a scheme of the minmax PRVP viewed as a BAP.

![Figure 1: Approaches for solving the minmax PRVP as a BAP](image)

### 3.2 The minmax PRVP as a particular case of the BAP

The minmax PRVP considered as a BAP has some specific properties presented below:

1. There is a possibility of having a good heuristic solution quickly at our disposal.
2. The matrix element values grow to the right and to the left from ideal positions.
3. It is easy to calculate an assignment matrix, and it is possible to avoid storage of a full matrix, only computing elements, for which \( |x_i - r_j| < I \) (“quota” band) is fulfilled, at each moment, if we are restricted by available memory.
4. There are always feasible solutions belonging to the “quota” band (for symmetric discrepancy functions).

From this it can be deduced that new approaches that combine and make use of such properties could be efficient with regard to solving the minmax PRVP as a BAP. The BAP matrix for the problem is composed of values that can be computed (Bautista et al. (1997)) as follows:

$$\phi_{ik}(t) = \max\left[f(k-1,t-1), f(k, t)\right]$$

For solving this problem with symmetric discrepancy functions it suffices to use the functions

$$f(k, t) = \left| k-r_i \cdot t \right| = \left| \frac{k \cdot U - u_i \cdot t}{U} \right|,$$

since for any other symmetric $f(k, t)$ the order of the matrix element values will kept the same. In this case, despite the original matrix values being rational, we can work with integer variables that enables us to achieve more rapid algorithm performance. In order to solve the problem with asymmetric discrepancy functions we have defined them as follows:

$$f(k, t) = \alpha_1 \cdot \max(0, k-r_i \cdot t) + \alpha_2 \cdot \max(0, r_i \cdot t-k)$$

with, in a particular case, $\alpha_1 = 1$ and $\alpha_2 = 3$, thus we penalize the underproduction more than the overproduction. The value of the objective function $z_m$ is given by the value corresponding to the maximal cost $\hat{\phi}_{ik}$ entering in the final solution:

$$z_m = \max_{1 \leq i \leq V} \max_{1 \leq k \leq U} \hat{\phi}_{ik}(t)$$

### 3.3 Computational experience

This section presents comparative computational results of application of newly designed and already known approaches to some instances of the PRVP, that are generated by fixing the total number of units to be produced and the product type number, and randomly selecting the product number of each type, which is uniformly distributed. All available codes from the literature and codes that we have additionally implemented are in FORTRAN. The algorithms have been executed on a SUN 450 Ultra SPARC2 with a processor of 250 Mhz and 512 of RAM.

To perform computational experiments we have chosen BI-APS (with LAPJVsp), MPS and BI-MPS approaches designed for the minmax PRVP solution, which seem to be of rapid performance, and two specific BAP algorithms. The time required for any matrix computation has not been taken into account as it is a constant for fixed problem dimension and the available memory permits storage even of a complete matrix. We have run BI-APS, MPS, BI-MPS, BASS and BOTJV algorithms with 100 instances randomly generated for the fixed product type number $V=50$ and the total unit number $U=2,000$. We have omitted computing average time since it is difficult to draw conclusions with regard to it, so computational results of the five chosen approaches for all (100) instances are presented in Figure 2. With regard to the BASS algorithm we have not included the computational time results for the instances 21-25, 84-86, 89-93, 96-100 because of being too high in comparison with results provided by other approaches.
Figure 2: Time comparison for the set of 100 instances with \( U=2.000 \) and \( V=50 \)

According to our computational experiments the most rapid approach is that which uses BMCM algorithm (MPS or BI-MPS). The method with specific assignment problem algorithm LAPJVsp and bisection search technique (BI-APS) could be efficient with a low number of product types. However, the computational time increases with the augmentation of this number since the sparse matrix dimension is larger for the same fixed number \( U \). We have also observed that BASS, specific for BAP algorithm, always performs more quickly than BOTJV excepting the cases where input data has some specific structure (where the major part of a total demand belongs to one particular product type \( i \)). For such instances BOTJV algorithm runs exceptionally faster than for other and quite the opposite to the BASS algorithm whose performance time has grown surprisingly.

We have extended the initial computational experience for larger dimensions (up to \( V=5.000 \) and \( U=10.000 \)) applying the best performing approaches MPS and BI-MPS. Finally all methods have been tried with asymmetric discrepancy functions. With regard to the computational time in general its augmentation has been noted. Table 1 presents an example of all methods’ application to the particular instance with symmetric and asymmetric discrepancy functions \( (\alpha_1 = 1 \text{ and } \alpha_2 = 3) \).

<table>
<thead>
<tr>
<th>( V=40, U=1000 )</th>
<th>APS</th>
<th>BI-APS</th>
<th>MPS</th>
<th>BI-MPS</th>
<th>BOTJV</th>
<th>BASS</th>
<th>Bottleneck:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>6.59 (23)</td>
<td>1.93 (6)</td>
<td>0.18 (7)</td>
<td>0.17 (5)</td>
<td>1.34</td>
<td>0.86</td>
<td>0.922</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>93.72 (64)</td>
<td>13.73 (9)</td>
<td>0.16 (4)</td>
<td>0.54 (12)</td>
<td>2.67</td>
<td>3.21</td>
<td>2.766</td>
</tr>
</tbody>
</table>

Table 1: Example with symmetric and asymmetric discrepancy functions

4. Conclusions

In this research the \textit{minmax} PRVP of any dimensions regarded as a BAP has been solved successfully. The computational experience of the \textit{minmax} PRVP solution for small dimensions and, for the first time, for large dimension instances has been
obtained. New efficient methods, which allow dealing with such instances, have been devised. According to our computational experience, approach using a sequence of matching problems to solve the minmax PRVP proposed by Steiner and Yeomans (1993) and improved by introduction of a new initialization outperforms computational results of others for the majority of the tried instances. We have observed that the BI-MPS (with bisection search) may or may not perform better than the MPS for the instances of a special structure, while for the rest of instances there is not almost any change with relation to the computational time. Finally, for the tried instances the BI-MPS approach with bisection search appears to perform worse than the MPS approach when the objective function is asymmetric. To sum up, suppose the minmax PRVP instance has to be solved. The MPS or BI-MPS approaches should be applied since they seem to be the most efficient among the proposed in the present research.

References


